REDUCED ORDER MODELING OF FLUID STRUCTURAL INTERACTIONS IN MEMS BASED ON MODAL PROJECTION TECHNIQUES

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ABSTRACT

Reduced order macromodeling (ROM) of electrostatic structural interaction has become state of the art for fast component and system simulations. The following paper presents a new approach to add dissipative effects into existing macromodels for damped harmonic and transient analyses. The models are automatically generated by a modal projection technique based on the harmonic transfer functions of the fluidic domain. The transfer functions are either obtained at the initial position (small signal case) or at various deflection states (large deflection case). This method has been successfully applied to squeeze and slide film problems and holds for nontrivial plate shapes and arbitrary motion.

INTRODUCTION

MEMS designers still long for efficient means to describe the behavior of complex electromechanical systems. Difficulties arise from the fact that various physical phenomena act on the same part of the structure and can't be considered autonomously. For example the movable mechanical component of most microstructures serves simultaneously as a plate of a capacitive arrangement and as a moving wall of a compressed fluid. As a consequence the governing partial differential equations become coupled and, if the structure undergoes large displacements, non-linear. There has been enormous progress in solving such problems with the introduction of reduced order modeling techniques [1-2].
Recent contributions have shown that electromechanical systems can efficiently be described by modal superposition methods. Essential speed up for the structural analysis is achieved since the deformation state of the mechanical system is represented by a weighted combination of a few eigenmodes. Capacitance-stroke functions provide non-linear coupling between the modes and the electrical quantities, such us electrostatic forces and electrical current, if stroke is understood as modal amplitude. Modal representations of MEMS are very efficient since just one equation per mode and one equation per conductor are necessary to describe the electromechanical system entirely.
A bottleneck of reduced order modeling techniques is still dissipative effects such as viscose damping in the surrounding fluid. It is still unclear how to reduce the true dynamic nature of the general Navier-Stokes equation of fluidic systems and represent the results in modal coordinates. Feasible are systems where the fluid-structural interactions can be described by squeeze or slide film equations which cover a very important class of MEMS. The following article demonstrates an automated procedure to extract squeeze film macromodels from a few finite element runs.

THEORETICAL BACKGROUND

A. SOLVING SQUEEZE FILM PROBLEMS

Reynold's squeeze film equation is applicable for structures where a small gap between two plates opens and closes with respect to time [3]. This assumption holds for structures where the seismic mass moves perpendicular to a fixed wall, for plates which tilt around horizontal axes and for clamped beams where the flexible part moves against a fixed wall.
Eq. (1) relates the pressure change p in the fluid gap for any given wall velocity v_z

$$\frac{d^3}{12\,\eta_{eff}}\left(\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}\right)=\frac{d}{p_0}\frac{\partial p}{\partial t}+v_z \qquad (1)$$

where p_0 is the ambient pressure, d the local gap separation and η_{eff} the effective dynamic viscosity. Note that d and η_{eff} might also by functions of x and y in case of non-uniform gap separation. The computed pressure results will later be used to extract characteristic damping parameter.
A special treatment is necessary to consider the flow resistance at perforation holes. Viscose fluid link elements based on the theory of capillary flow where introduced to model the gas exchange from the gap to the open space or to a second gap which might occur in case of encapsulated devices. The hole resistance R_{CH} is [4]

$$R_{CH}=\frac{8\,\eta^*_{eff}\,(1+\Delta\Gamma\,)L}{r^4\pi} \qquad (2)$$

where η^*_{eff} is the effective hole viscosity which captures size effects, r the radius, L the hole length and $\Delta\Gamma$ the channel length extension to consider inlet and outlet effects. Fig. 1 shows a typical finite element model with structural and fluid elements of a clamped-clamped beam example.

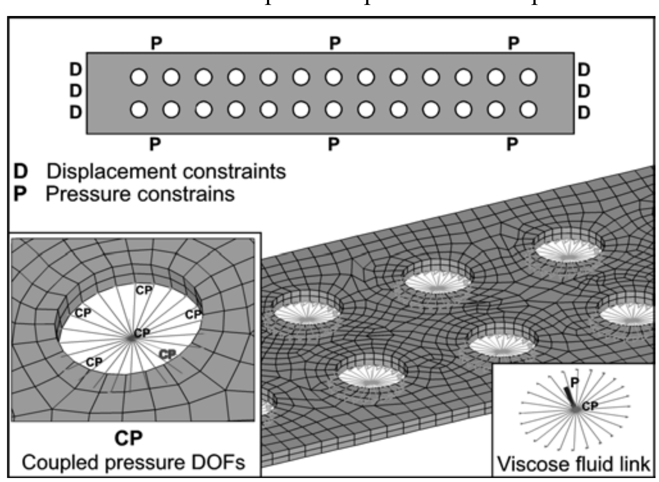

Figure 1. Finite element modeling of the fluid domain

B. MODAL PROJECTION TECHNIQUES

Any motion of the structural domain which has non-zero components in the z-direction causes a pressure change in the gas film. At low frequencies the pressure change varies with respect to the structural velocity and acts as damping. At higher frequencies it affects the stiffness matrix due to the fluid compression. The ultimate goal of the ROM procedure is to find an equivalent damping and stiffness matrix representation which captures the true dependency between structural velocities v and fluid pressure $p(v)$ but is written in modal coordinates.

The coefficients C_{ji} and K_{ji} of such a matrix representation can be obtained from the following modal force balance equation

$$C_{ji}\ \dot{q}_i + K_{ji}\ q_i = \phi_j^T \int N^T p\left(\phi_i\, \dot{q}_i\right) dA\,. \qquad (3)$$

where q_i is the modal coordinate, ϕ_i is the i^{th} eigenvector and N^T the vector of the finite element shape functions. C_{ji} and K_{ji} state the dependency between structural wall velocities *caused by mode i* and the reacting fluid forces which *act on mode j*. The damping and squeeze coefficients of each mode can be found on the main diagonal terms. Off-diagonals are responsible for the fluidic cross-talk among modes which happens in case of asymmetric gap separation (Fig. 2).

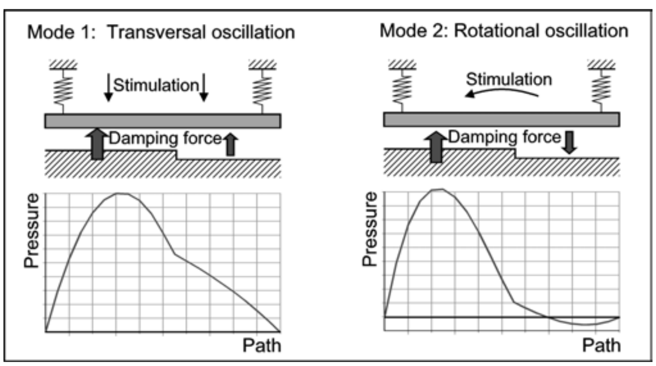

Figure 2. Modal interactions due to an asymmetric gap

The following algorithm must be performed to compute all coefficients of C and K (see Fig. 3):

1. Excite the squeeze film model by wall velocities which are equal to the values of the first eigenvector.
2. Perform a harmonic response analysis to compute the pressure response in the entire frequency range.
3. Integrate the real and imaginary part of the element pressure and compute the complex nodal force vector for each frequency.
4. Compute the scalar products of *all* eigenvectors and the nodal force vector of step 3. The resulting numbers are modal forces and state how much of the pressure distribution acts on each mode (back-projection).
5. Extract the damping and squeeze coefficients from the real and imaginary part of the modal forces.
6. Repeat step 1 with the next eigenmode.

A modal decomposition of damping effects becomes admissible since the Reynold's squeeze film equation is linear. It is possible to assess the damping properties of individual modes for a unit velocity according Eq. (3) and later, when using the ROM, we superimpose several modes and scale to the current 'or true' velocities. In contrast to the quasi-static finite element simulations, which are necessary to extract capacitance information of electrostatic-structural interactions, the new approach requires a series of harmonic response analyses to capture the dynamic nature of fluid-structural interactions.

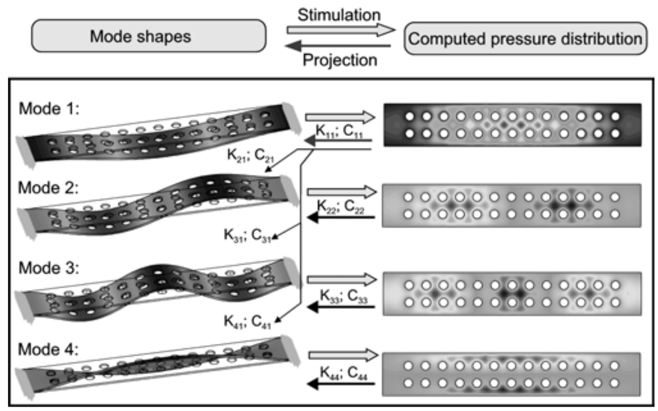

Figure 3. Modal projection of fluidic forces on modes

It seams to be burdensome to evaluate the entire frequency spectrum for damping data extraction. Fortunately, the harmonic transfer functions of squeeze film problems are very smooth and can be interpolated from a few data points. In practice we analyze just five frequencies which are regular spaced around the cut-off frequency of each mode.

C. MACROMODEL CONSTRUCTION

Frequency dependent matrix coefficients are not applicable for transient simulations since frequency as a primary variable does not exist. Veijola [4] proposed a very interesting approach of representing damping data by a network of frequency independent spring-damper elements which leads to the same harmonic transfer function (Fig. 4). The unknown parameters $\widetilde{C}_i$ and $\widetilde{K}_i$ of such a network are determined automatically by a least square fit

$$C(\Omega) = \sum_{i=1}^{Level} \frac{\widetilde{C}_i^{\,-1}}{\widetilde{C}_i^{\,-2} + \Omega^2\ \widetilde{K}_i^{\,-2}} \qquad (4)$$

$$K(\Omega) = \sum_{i=1}^{Level} \frac{\Omega^2\ \widetilde{K}_i^{\,-1}}{\widetilde{C}_i^{\,-2} + \Omega^2\ \widetilde{K}_i^{\,-2}} \qquad (5)$$

where Ω is the circular response frequency.

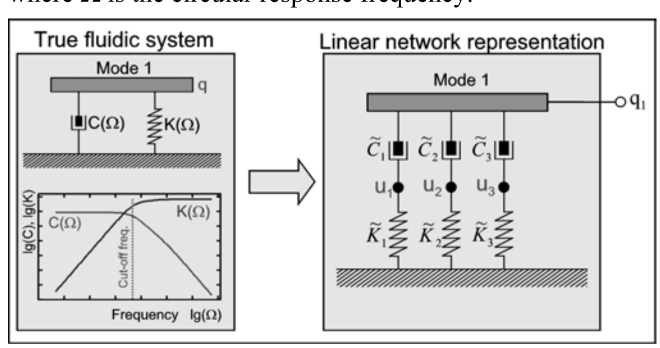

Figure 4. Network representation of transfer functions

A schematic view on the network representation of two modes is shown in Fig. 5. Special precautions are necessary to consider the correct sign of modal interactions. Inherently, spring-damper elements will always generate forces of the same direction. For example, if mode 1 moves in a positive direction then mode 2 wants to go in the same direction and vice versa.

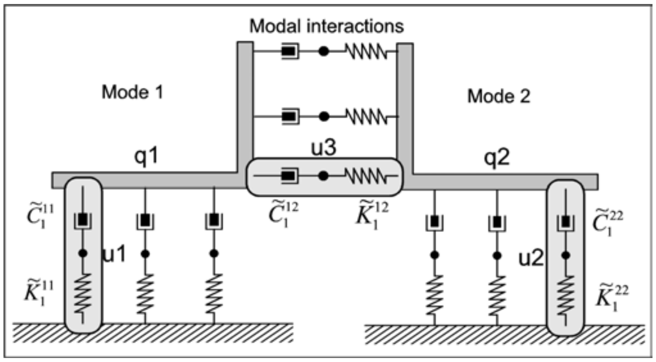

Figure 5. Schematic view on the network of two modes

In practice, the computed modal interactions might be different. A positive motion of mode 1 can also cause fluidic forces which drive mode 2 in a negative direction. Hence we obtain negative numbers for the spring and damper elements. If one establishes the final matrix equation system and transfers the negative value of the coupling terms into the governing equations, the solution will be wrong.

A stable numerical approach to capture the correct dependency of interacting modes is given in Eq. (6). The following equation system covers the highlighted elements of Fig. 5 for simplification.

$$\left[\begin{array}{cc|cc|c} \widetilde{C}_1^{11}-\left|\widetilde{C}_1^{12}\right| & -\widetilde{C}_1^{11} & 0 & 0 & \widetilde{C}_1^{12} \\ -\widetilde{C}_1^{11} & \widetilde{C}_1^{11} & 0 & 0 & 0 \\ \hline 0 & 0 & \widetilde{C}_1^{22} & -\widetilde{C}_1^{22} & 0 \\ 0 & 0 & -\widetilde{C}_1^{22} & \widetilde{C}_1^{22} & 0 \\ \hline \widetilde{C}_1^{12} & 0 & 0 & 0 & -\left|\widetilde{C}_1^{12}\right| \end{array}\right] * \left[\begin{array}{c} \dot{q}_1 \\ \dot{u}_1 \\ \hline \dot{q}_2 \\ \dot{u}_2 \\ \hline \dot{u}_3 \end{array}\right] +$$

$$\left[\begin{array}{cc|cc|c} 0 & 0 & 0 & 0 & 0 \\ 0 & \widetilde{K}_1^{11} & 0 & 0 & 0 \\ \hline 0 & 0 & -\left|\widetilde{K}_1^{12}\right| & 0 & \left|\widetilde{K}_1^{12}\right| \\ 0 & 0 & 0 & \widetilde{K}_1^{22} & 0 \\ \hline 0 & 0 & \left|\widetilde{K}_1^{12}\right| & 0 & -\left|\widetilde{K}_1^{12}\right| \end{array}\right] * \left[\begin{array}{c} q_1 \\ u_1 \\ \hline q_2 \\ u_2 \\ \hline u_3 \end{array}\right] = \left[\begin{array}{c} F_1 \\ 0 \\ \hline F_2 \\ 0 \\ \hline 0 \end{array}\right] \quad (6)$$

Effects of modeling modal interactions can be determined for systems with asymmetric gap separation as that shown in Fig. 2. The transversal oscillation mode was stimulated by a force jump and the transient response of mode 1 and mode 2 are observed (see Fig. 6). As expected, the direction of mode 2 changes if the smaller gap was flipped to the right. Similar results could be obtained in a harmonic response analysis. The amplitudes are independent from the gap orientation but there is a phase shift of 180° between both models.

Practical experiences have shown that interactions among high order modes are usually negligible and that the network representation of higher modes can be simplified to a single spring damper element.

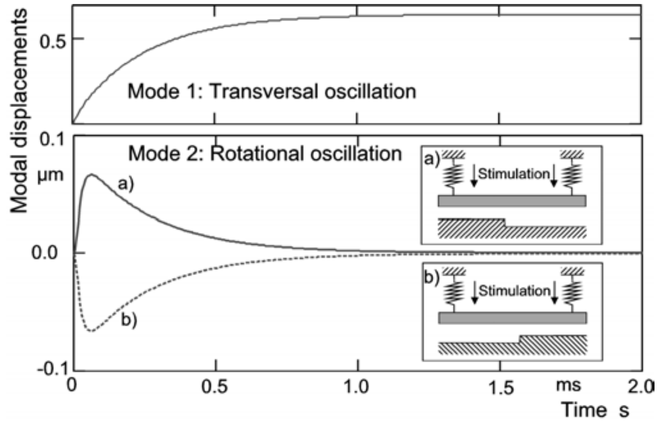

Figure 6. Transient response after a force jump

D. LARGE DISPLACEMENT MODELING

Large displacement damping models become important if the gap separation changes significantly during operation. One should keep in mind that the Reynold's equation is linearized and describes dissipative effects at the initial position. A repeated use of the numerical squeeze film analyses, similar to the electrostatic-structural data sampling, allows us to gather damping information at various points of operation. The behavior between data points can be interpolated by analytical functions (response surface) which are obtained by a least square fit.

To obtain a good quality of the response surface it is vital to chose a proper trial function. Polynomials are very popular because the unknown coefficients are easy to find and local derivatives can be computed with less effort. Since the damping and squeeze functions have poles but no zeros over the entire operating range we prefer inverted polynomials instead of the more complicated rational polynomials.

The trail function for the multivariable polynomial is

$$\widetilde{C}(\vec{q}) = \left(\sum_{i1=1}^{Ord1} \sum_{i2=1}^{Ord2} \cdots \sum_{im=1}^{Ordm} a_{i1} a_{i2} \cdots a_{im} q_1^{i1} q_2^{i2} \cdots q_m^{im} \right)^{-1} \quad (7)$$

where a_i are the unknown polynomial coefficients and Ord_i the order in each dimension.

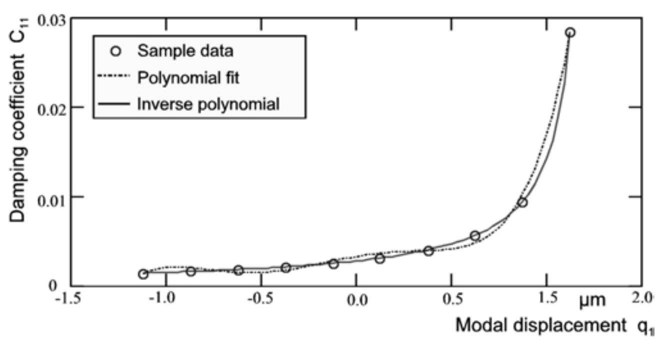

Figure 7. Response function for the large signal case

Small and large signal capabilities of the discussed damping models are implemented in a beta-test element ROM140 in the ANSYS program. It allows up to nine shape functions and can directly be attached to the existing electrostatic-structural ROM144 element [2] for harmonic and transient coupled domain simulations (see Fig. 8).

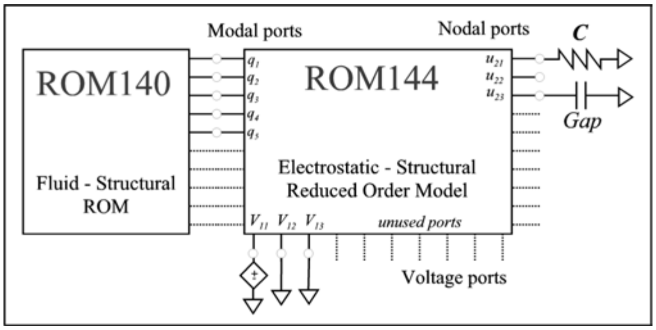

Figure 8. ROM model for coupled domain simulations

SIMULATION RESULTS

The coupled domain model of Fig. 8 was used to simulate the transient response of several RF-microswitches developed at the DaimlerChrysler Research Center, Ulm [5]. One of these structures is shown in Fig. 9. It consists of a double sided clamped beam with an underlying center electrode. Perforation holes of different diameter and distance were manufactured in order to optimize the device performance.

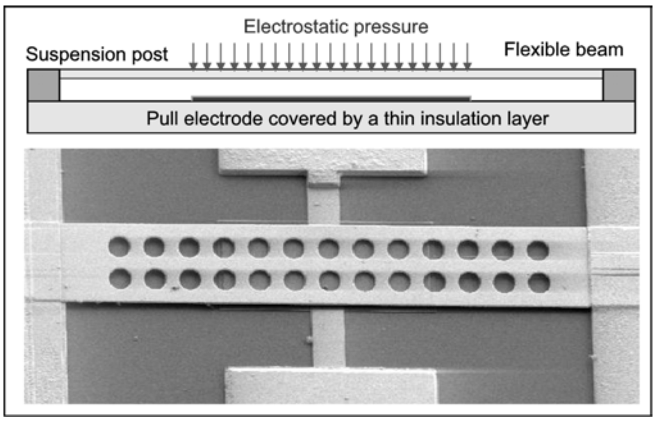

Figure 9. SEM photography of a RF-microswitch

Reduced order models are very powerful in analyzing the static and dynamic behavior of coupled domain systems. Essential design parameters such as pull-in and hystereses release voltage, static voltage-deflection functions, mechanical stress, eigenfrequencies and the switching and release time can be analyzed within seconds (Table 1). Fig. 10 shows displacement time functions at the center node after a voltage jump. The transient results deviate less than 15% with the experimental data.

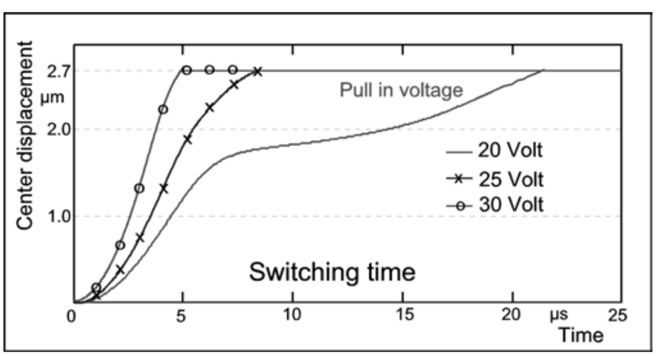

Figure 10. Transient response after a voltage jump

Table 1: Summary of computation times (Clock speed 800 MHz)

	Type A	Type B
Number of Elements		
Structural solids	2382	1218
Electrostatic solids	1287	853
Fluid shell (viscose link)	1192 (26)	609 (10)
ROM Generation Pass		
Electrostatic-structural	47 min	35 min
Fluid-structural (small signal case)	≈5 min	≈4 min
Electrostatic-fluid-structural (large signal case)	135 min	92 min
ROM Use Pass		
Voltage sweep (50 steps)	8 sec.	6 sec.
Harmonic response (500 substeps)	14 sec.	10 sec.
Transient analysis (850 time steps)	37 sec.	22 sec.

CONCLUSIONS

The fluid-structural domain of a coupled-field FE model can be reduced in its DOF number by several decades retaining the three-dimensional distributed characteristic of its interactions with the structural domain. Harmonic transfer functions and the transient response can be calculated without significant rigid-body simplification by desktop computational power.
Extensions of the presented squeeze film models by full Navier-Stokes equations are suggested for future work. This requires a network of spring-mass-damper elements for the fluidic macromodel. A much larger number of practical cases can then be satisfied.

Acknowledgments

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References

[1] J.E. Mehner, L.D. Gabbay, and S.D. Senturia: Computer-Aided Generation of Nonlinear Reduced-Order Dynamic Macromodels, J. Microelectromech. Syst., Vol. 9 No. 2, 270-277, 2000
[2] ANSYS 7.0 Documentation, ANSYS Inc., Canonsburg PA. USA 2002
[3] J.E. Mehner, S. Kurth, D. Billep, et.al.: Simulation of Gas Film Damping on Microstructures with Nontrivial Geometries, Proc. of the MEMS Conference, Germany, 1998
[4] T. Veijola: End effects of rare gas flow in short channels and squeezed-film dampers, MSM 2002, 104-107, San Juan 2002
[5] B. Schauwecker, K.M. Strohm, J.E. Mehner, et.al. : Toggle-Switch - A new type of RF MEMS switch for power applications, IMS 2002, Vol. 1, 219-222, Seattle, USA